§54. Study on Nonequilibrium Statistical Mechanics of NS and MHD Turbulence by Using Massive DNS

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We have studied the statistical properties of the incompressible Navier-Stokes turbulence and passive scalar convected by the turbulent flow. Schmidt number $Sc = \nu/\kappa$ is unity. The fields are assumed to be periodic in three directions and expanded in the Fourier series. The 4th order Runge-Kutta-Gill method is used for time integration and the Gaussian white random force (source) for the NS (passive scalar) equation is applied at lower end of the wavenumber band. Key to the massive DNS of the turbulence is to develop an efficient 3D FFT for NEC-SX7. Our strategy of designing the program is to write the NS solver in terms of HPF for easiness to handle and to write 3D FFT in MPI for efficiency. Currently the 3D FFT written in HPF is under operation and the radix-4 FFT rather than the radix-2 FFT is developed by considering the latency of SX7. It is found that the radix-4 FFT is more efficient than the radix-2 FFT when the grid number is large.

Maximum number of grid points is $N = 1024^3$. Achieved Reynolds and Péclet numbers are Run 1: $(\mathcal{R}_\lambda, Pe_\lambda) = (258, 145)$, Run 2: $(435, 235)$, respectively.[2] Statistical average is taken as time average and the period is $T_{av} = 6T_{eddy}$ for Run 1, $0.5T_{eddy}$ for Run 2, where T_{eddy} is a large scale eddy turn over time.

The spectrum for the scalar variance is defined by

$$\langle \theta^2 \rangle / 2 = \int_0^\infty E_\theta(k) dk.$$

When the intermittency effect is neglected, $E_\theta(k)$ is

$$E_\theta(k) = C_{OC} \bar{\chi} \epsilon^{-1/3} k^{-5/3}$$

in the inertial-convective range, where C_{OC} is Obukhov-Corrsin constant, and $\bar{\chi}$ is the average rate of the scalar dissipation. Figure 1 shows the compensated spectra for the kinetic energy and scalar variance. Kolmogorov constant is found to be $K = 1.64 \pm 0.02$ and $C_{CO} = 0.66 \pm 0.04$.[2] Both values are in good agreement with the experimental values $K_{exp} = 1.62, C_{OC,exp} = 0.68$.

The qth order moments of the longitudinal velocity increment $\delta u_r = u(\boldsymbol{x}+r\boldsymbol{e}_x) - u(\boldsymbol{x})$, the scalar increment $\delta\theta_r = \theta(\boldsymbol{x}+r\boldsymbol{e}_x) - \theta(\boldsymbol{x})$, and the mixed one are defined as

$$S_q^L(r) = \langle |\delta u_r|^q \rangle, \qquad S_q^\theta(r) = \langle |\delta\theta_r|^q \rangle,$$

$$S_q^{\theta L}(r) = \left\langle |(\delta\theta_r)^2 \delta u_r|^{q/3} \right\rangle.$$

The local scaling exponents $\zeta_q^\alpha(r) = d \log S_q^\alpha(r) / d \log r$, $(\alpha = L, \theta, \theta L)$ are computed as functions of the separation r. Their values in the inertial-convective and the viscous-convective ranges for various order q are shown in Fig.2. It is found that the scaling exponents are smaller than those of the velocity field, which means that the intermittency of the scalar field is stronger than that of the velocity field.

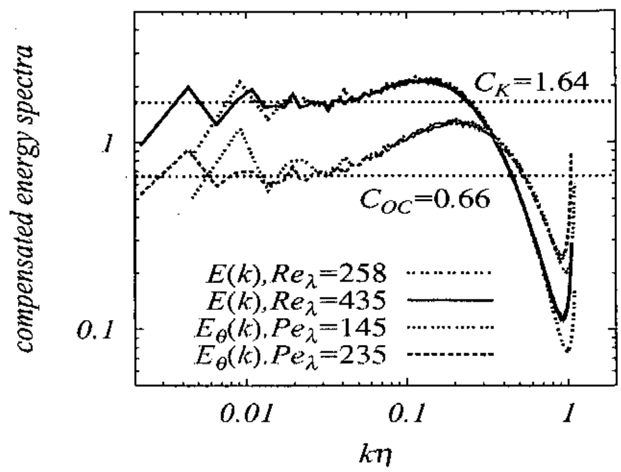

Fig.1 Compensated spectra of the kinetic energy and scalar variance. $\bar{\epsilon}^{-2/3}(k/k_d)^{5/3}E(k)$, $\bar{\chi}^{-1}\bar{\epsilon}^{-1/3}(k/k_d)^{5/3}E_\theta(k)$.

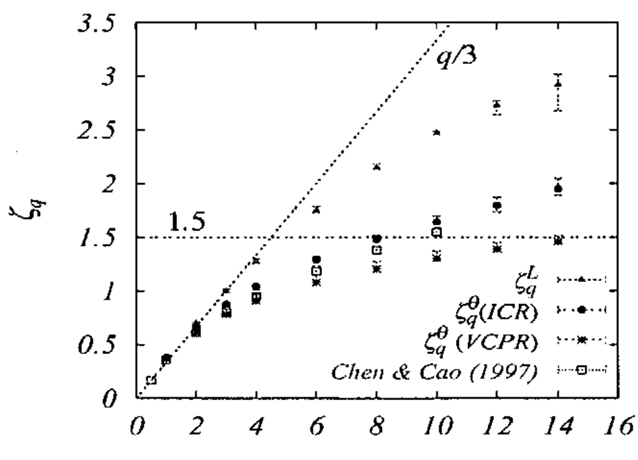

Fig.2 Comparison of the scaling exponents.

1) T. Gotoh, D. Fukayama, and T. Nakano, Phys. Fluids **14**, (2002) 1065.

2) T. Watanabe and T. Gotoh, New J. of Phys., (2004). http://www.iop.org/EJ/abstract/1367-2630/6/1/040/